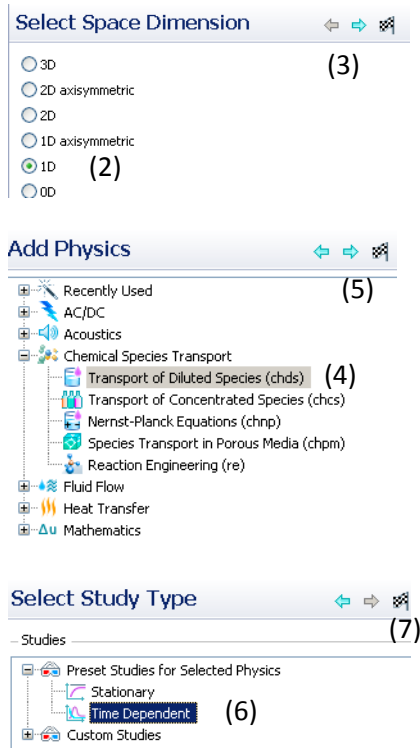
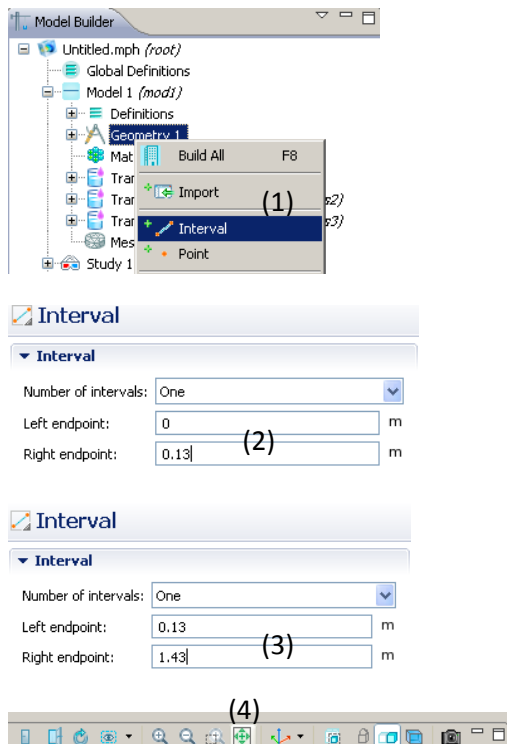


## Problem Type Specification



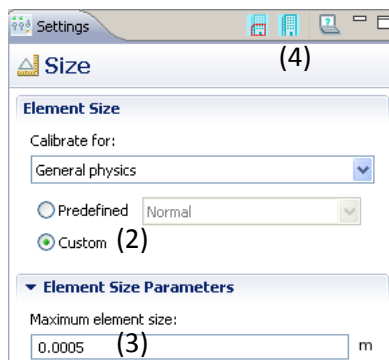
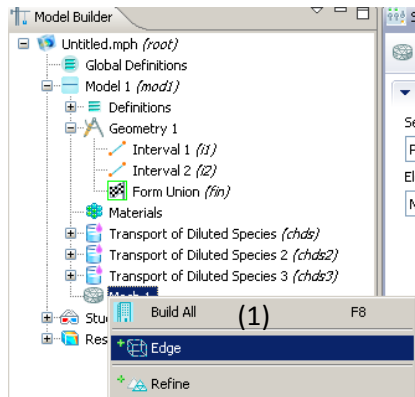
- (1) Open COMSOL 4.1 and save file. Remember to save file often.
- (2) Under **Select Space Dimension**, select 1D.
- (3) Click on the blue next arrow to add physics.
- (4) Double click “Transport of Diluted Species (chds)”. Repeat two more times for a total of three species (c, c2, c3).
- (5) Click on blue next arrow.
- (6) Select “Time Dependent”.
- (7) Click checked flag to finish.

## Geometry Creation



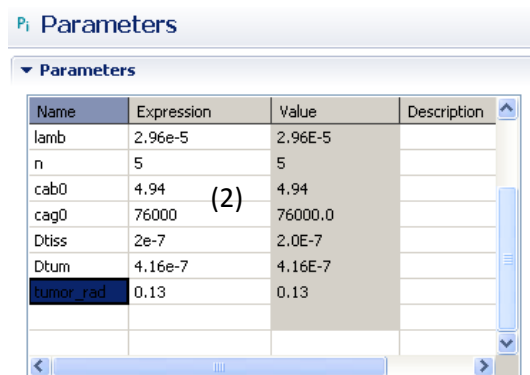
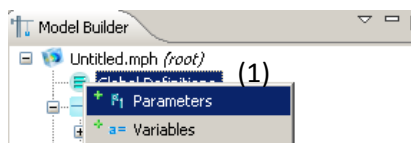
- (1) Right click on “Geometry 1” and select to add new “Interval”. Repeat to create two intervals.
- (2) Click on “Interval 1”. Input 0 m for left endpoint and 0.13m for right endpoint. This represents the tumor.
- (3) Click on “Interval 2” and input 0.13m for left endpoint and 1.43 for right endpoint. This line represents the tissue. Click on the “Build All” icon.
- (4) Under **Graphics**, click on the green expand icon to fit geometry to screen.

## Meshing



- (1) Right Click on “Mesh 1” and select to add new “Edge” meshing type.
- (2) Click on “Size” from “Mesh 1”. Under Element Size, click bubble for “Custom”. This will allow you to modify the meshing parameters.
- (3) Under Element Size Parameters, input 0.0005 m for Maximum element size.
- (4) Click the blue build all icon. The mesh should now have 2860 edge (mesh) elements.

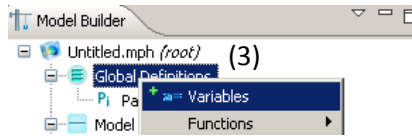
## Governing equations, source terms, I.C., B.C.



Constants:

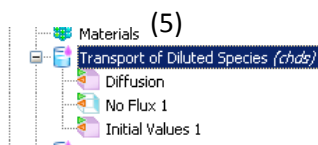
- (1) In **Model Builder**, right click on “Global Definitions” and select to add new list of “Parameters”.
- (2) Define the following parameters

Name	Expression
kbl	4.60e-5
kly	1.78e-5
kp1	5.00e-5
km1	1.00e-5
lamb	2.96e-5
n	5
cab0	4.94
cag0	76000
Dtiss	2.00e-7
Dtum	4.16e-7
tumor_rad	0.13



**Variables**

Name	Expression	Unit
Dtum_r	Dtum*x*x (4)	m <sup>2</sup>
Dtiss_r	Dtiss*x*x	m <sup>2</sup>
Rtiss_r	(kbl*cab0*exp(-lamb*t)-kly*mod1.c)*x*x	
Rtum_r	(-kp1*mod1.c*mod1.c2+km1*mod1.c3)*x*x	
Rag_r	n*(-kp1*mod1.c*mod1.c2+km1*mod1.c3)*x*x	
Rcm_r	((kp1*mod1.c*mod1.c2)-(km1*mod1.c3))*x*x	
A_r	x*x	m <sup>2</sup>



**Transport of Diluted Species**

Interface Identifier: chds

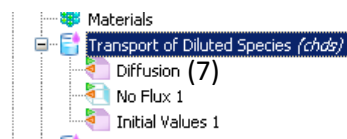
Domains: Selection: All domains

Equation: Transport Mechanisms

Additional transport mechanisms:

Convection (6)

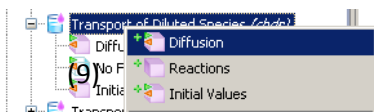
Migration in electric field



**Diffusion**

Diffusion coefficient: (8)

$D_c$  Dtum\_r m<sup>2</sup>/s



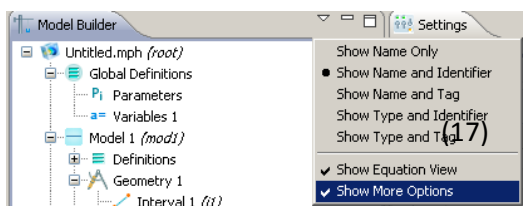
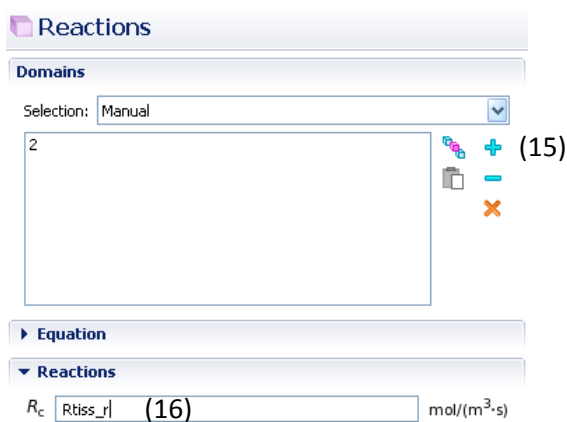
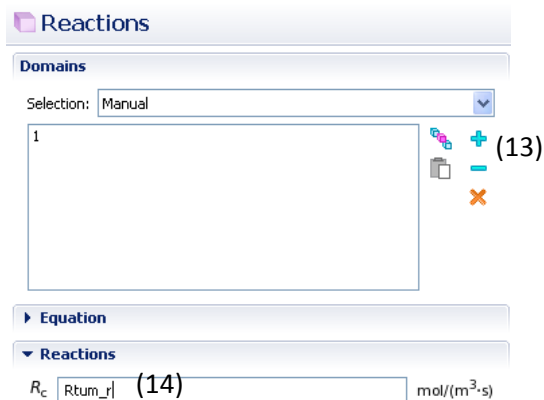
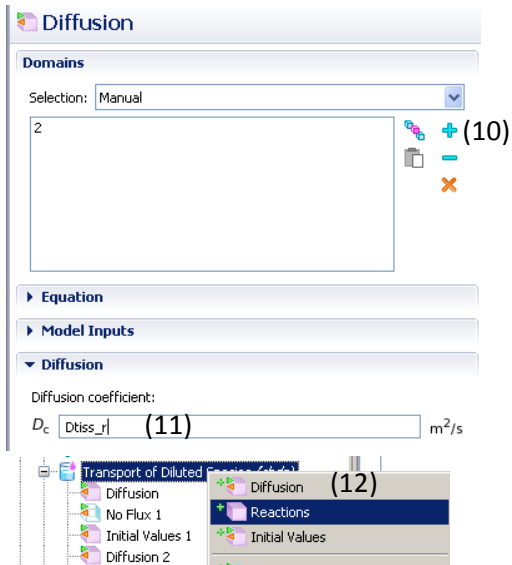
Expressions:

- Right click on "Global Definitions" and select to add new list of "Variables"
- Define the following expressions

Name	Expression
A_r	x*x
Dtum_r	Dtum* x*x
Dtiss_r	Dtiss* x*x
Rtiss_r	(kbl*cab0*exp(-lamb*t)-kly*mod1.c) * x*x
Rtum_r	(-kp1* mod1.c* mod1.c2+km1* mod1.c3) * x*x
Rag_r	n*(-kp1* mod1.c* mod1.c2+km1* mod1.c3) * x*x
Rcm_r	(kp1* mod1.c* mod1.c2-km1* mod1.c3) * x*x

Species 1: antibody

- Expand and click on "Transport of Diluted Species (chds)".
- Under Transport Mechanisms, uncheck box for "Convection".
- Under "Transport of Diluted Species (chds)" click "Diffusion".
- Set Diffusion coefficient to Dtum\_r.
- Right click on "Transport of Diluted Species (chds)" and add new "Diffusion" parameter.



- (10) Under **Graphics**, select domain 2 and click the blue add icon.
- (11) Set diffusion coefficient to  $D_{tiss\_r}$ .
- (12) Right click on “Transport of Diluted Species (chds)” and select to add new “Reactions”. Repeat to have two new “Reactions” parameter.
- (13) Click on “Reactions 1” and click on subdomain 1 in the **Graphics** window and click blue add icon.
- (14) Input  $R_{tum\_r}$  under Reactions.
- (15) Click on “Reactions 2” and click on subdomain 2 in the **Graphics** window and click blue add icon.
- (16) Input  $R_{tiss\_r}$  under Reactions.
- (17) On the **Model Builder** title bar, click on the upside-down triangle to open the drop down menu. Click on “Show Equation View” and “Show More Options” to activate both actions.

**Transport of Diluted Species**

Interface Identifier  
Identifier:

Domains  
Selection: All domains

1  
2

Equation

Transport Mechanisms

Additional transport mechanisms:

Convection

Migration in electric field (18)

Consistent Stabilization

Streamline diffusion

Crosswind diffusion

Transport of Diluted Species (chds)

Diffusion

Equation View (19)

No Flux 1

Initial Values 1

Diffusion 2

Reactions 1

Reactions 2

**Equation View**

Variables

Name	Expression	Unit
chds.Dxx_c	Rtum_r	m
chds.Dyx_c	0	m
chds.Dzx_c	0	m
chds.Dxy_c	0	m
chds.Dyy_c	Rtum_r	m
chds.Dzy_c	0	m

Shape Functions

Name	Shape function	Unit	Description
c	Lagrange	mol/m <sup>3</sup>	Concentration

Weak Expressions (20)

Weak expression
-A_r*ct*test(c)-chds.Dxx_c*cx*test(cx)

(18) Click on “Transport of Diluted Species (chds)”.

Under Transport Mechanisms, uncheck box for “Convection”.

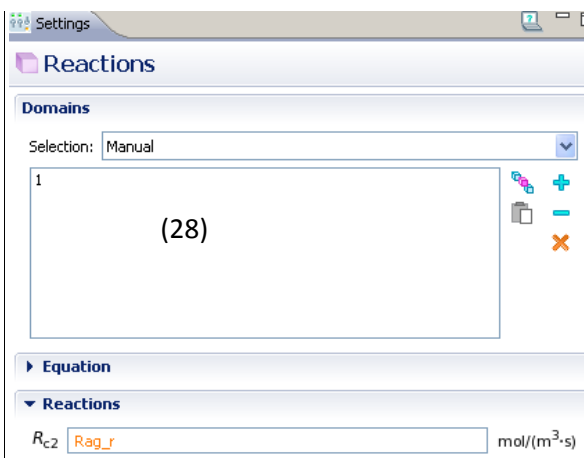
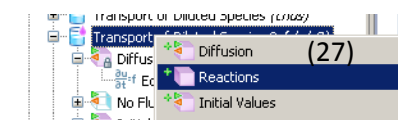
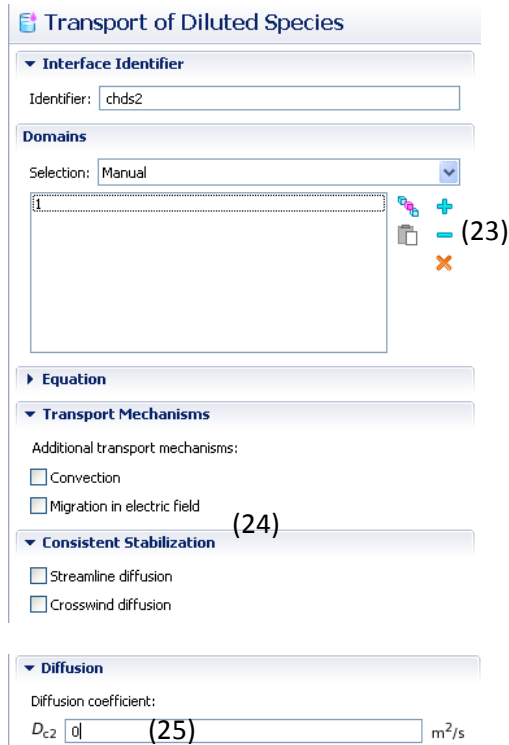
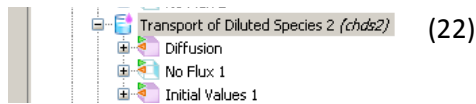
Under Consistent Stabilization, uncheck boxes for “Streamline diffusion” and “Crosswind diffusion”.

(19) Under “Transport of Diluted Species (chds)”, expand “Diffusion” and click on “Equation View”.

(20) In **Equation View**, under Weak Expression, multiply the existing equation with A\_r to result with

$-A_r \cdot c \cdot \text{test}(c) - \text{chds.Dxx}_c \cdot c_x \cdot \text{test}(c_x)$ .

(21) Repeat steps (19) and (20) for “Diffusion 2”.



Species 2: antigen

(22) Click on “Transport of Diluted Species 2 (chds2)”.

(23) In **Transport of Diluted Species** window, select domain 2 and click the minus icon to inactivate.

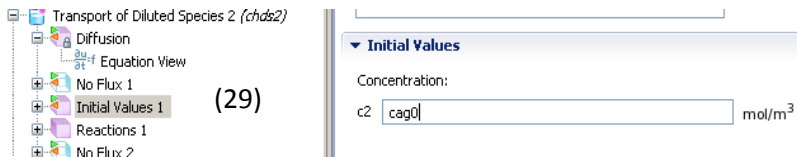
(24) Under Transport Mechanisms, uncheck box for “Convection”. Under Consistent Stabilization, uncheck boxes for “Streamline diffusion” and “Crosswind diffusion”.

(25) Select “Diffusion” from the **Model Builder** window and input 0 for the diffusion coefficient.

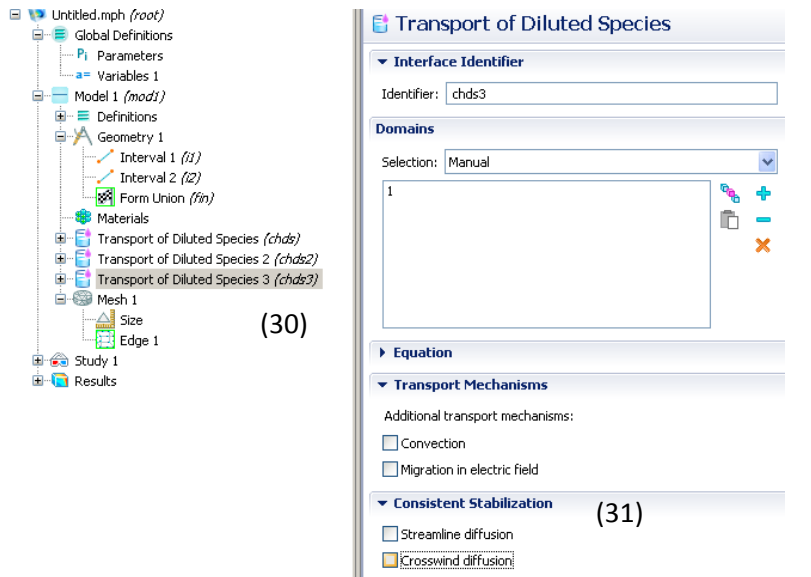
(26) Under “Diffusion” click on “Equation View”. In **Equation View**, multiply weak equation by with  $A_r$ .

(27) Right click on “Transport of Diluted Species 2 (chds2)” and add new “Reactions” parameter.

(28) Click domain 1 from **Graphics** window and click the blue add icon. This sets the reaction equation for just domain 1. Set Reactions to  $R_{ag\_r}$ .

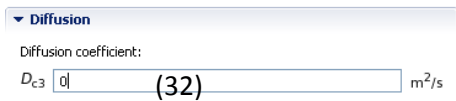


(29)

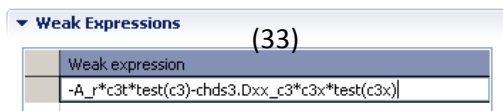


(30)

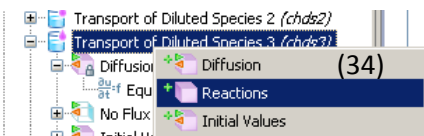
(31)



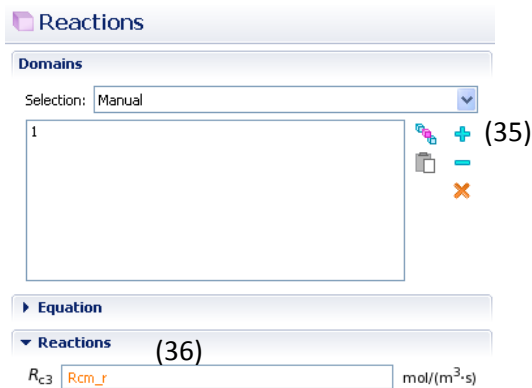
(32)



(33)



(34)



(36)

(35)

(29) Click on “Initial Values 1” and set to  $c_{ag0}$ .

Species 3: antibody-antigen complex

(30) Click on “Transport of Diluted Species 3 (chds3)”.

(31) In **Transport of Diluted Species** window, select domain 2 and click the minus icon. Under Transport Mechanisms, uncheck box for “Convection”. Under Consistent Stabilization, uncheck boxes for “Streamline diffusion” and “Crosswind diffusion”.

(32) Select “Diffusion” from the **Model Builder** window and input 0 for the diffusion coefficient.

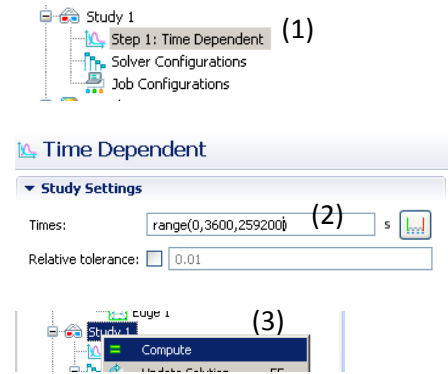
(33) In **Equation View**, multiply the weak expression by  $A_r$ .

(34) Right click on “Transport of Diluted Species 3 (chds3)” and select to add new “Reaction”.

(35) Click domain 1 from **Graphics** window and click the blue add icon. This sets the reaction equation for just domain 1.

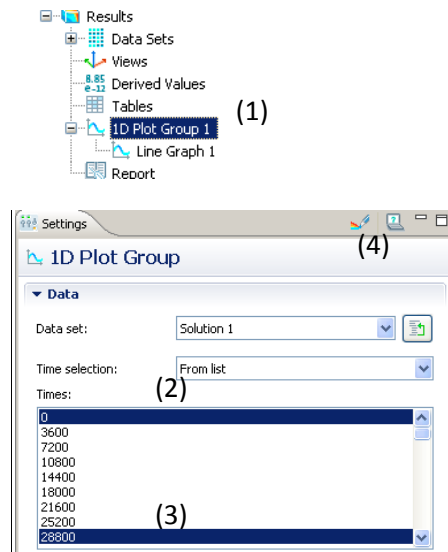
(36) Set Reactions to  $R_{cm\_r}$ .

## Solver Settings and Solution



- (1) Expand "Study 1" and select "Step 1: Time Dependent".
- (2) Set time range(0,3600,259200).
- (3) Right click "Study 1" and select "Compute" to solve the problem.

## Postprocessing



- (1) Click on "1D Plot Group 1".
- (2) Under **1D Plot Group**, select "From List" from the drop down menu of Time selection.
- (3) Hold the CTRL button and select the 6 desired times: 0, 28800, 57600, 172800, and 259200 in the Times area.
- (4) Click the plot icon (rainbow pencil).
- (5) The following concentration profile should result.

